

*

• •

Olesen

-

-

-

:

-

-

*

G_c :a : σ_c : -1

:Kic

Kic

$$\sigma < \sigma_c$$

Griffith

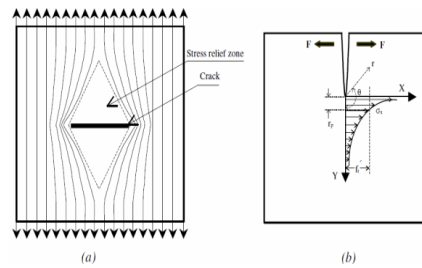
Griffith

NLFM

[6]

$$((1))$$

[6]1976 Hillerborg



(3) 2)

$$\sigma - W$$

a (1)

b

Shah & Jenk

(Kic)

(COD)

Irwin

Griffith

) RILEM

(

Griffith

$$\sigma_c = \sqrt{\frac{E G_c}{\pi a}} = \frac{K_{IC}}{\sqrt{\pi a}} \quad (1)$$

63

(

-3

$\sigma - w$

Ulfkjaer [1414]

$\sigma - w$

Ulfkjaer (4)

h

$$h = k \cdot d$$

$$k = 0.5$$

Olesen

.FPZ (Fracture Process Zone)

F_t

$\sigma - w$

w

w_c

(Kic)

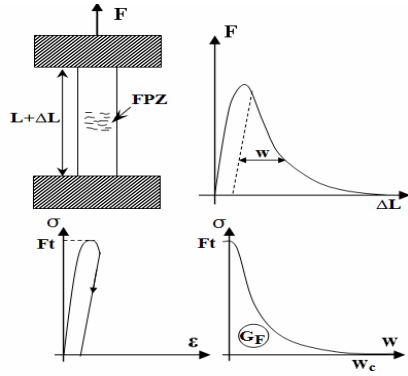
(CODc)

-1

-2

-3

-4



(2)

: -2

-5

(Kic)

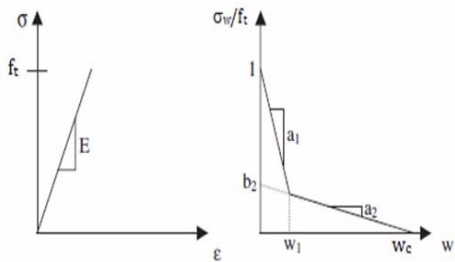
(CODc)

Olesen

[88]

(Kic)

(CODc)



(3)

()

$$\alpha = 1 - \beta_2 \frac{1-b_2}{4\theta} (2-\beta_2) - \sqrt{(1-\beta_2) \frac{(2-\beta_2)(1-b_2)^2}{(4\theta)^2(1-\beta_2)} - \beta_2 + \frac{(1-b_2)\beta_2}{2\theta} + \frac{b_2}{\theta}} \quad (10)$$

$$\mu = 4(1-3\alpha+3\alpha^2 - \frac{\alpha^3}{1-\beta_2})\theta + (6\alpha-3) \frac{(1-b_2)(\beta_2-\beta_1)\alpha^2}{1-\beta_2} \quad (11)$$

$$\theta_{II-III} = 0.5(\frac{b_2}{\beta_2} + \sqrt{\frac{\beta_2}{4}(1-b_2)^2 + \frac{1}{2}(1-b_2^2) + \frac{b_2^2}{\beta_2}}) \quad (12)$$

$$\alpha = 1 - \frac{1}{2\theta} (1 + \sqrt{1 - b_2 + \frac{b_2^2}{\beta_2}}) \quad (13)$$

$$\mu = 4(1-3\alpha+3\alpha^2 - \alpha^3)\theta + (6\alpha-3) - 3\alpha\alpha + \frac{b_2}{4\theta} (1 - \frac{b_2}{\beta_2})^2 - \frac{b_2(1-\alpha)}{2\theta} (1 - \frac{b_2}{\beta_2}) \quad (14)$$

$$M = \frac{PL}{4} + \frac{m_b \cdot g \cdot L}{8}$$

$$P = \frac{2\mu f_t h^2 b}{3L} - \frac{1}{2} m_b \cdot g \quad (15)$$

$$CMOD = COD + COD_g + COD_e \quad (16) \quad (1)$$

$$COD = \frac{sf_t}{E} \frac{1-b_i + 2\alpha\theta}{1-\beta_i}$$

$$(b_i, \beta_i) = \begin{cases} (1, \beta_1) & \text{Phase II} \\ (b_2, \beta_2) & \text{Phase III} \\ (0, 0) & \text{Phase IV} \end{cases} \quad (17)$$

$$COD_g = \frac{sf_t 2(a_0 + d)}{hE} (\theta - 1)$$

$$COD = \frac{6PL\alpha_0}{EbH^2} v1 \quad (2)$$

$$V1 = (V1(\text{Stang}) + V1(\text{Karihaloo})) \quad (3) [7][13]$$

$$V1(\text{Stang}) = \left(\frac{\alpha_0 + \alpha}{\alpha_0}\right) \cdot \left(3.87 \cdot y^2 - 2.04 \cdot y^3 + \frac{1.66}{(1-y)^2}\right) \quad (20)$$

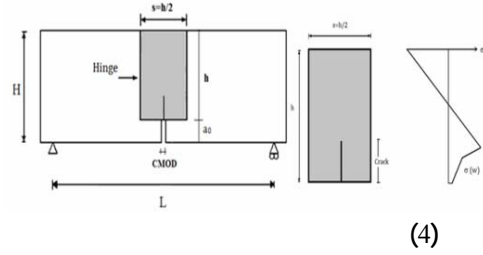
$$\sigma = F_t(b_i - a_i w) = \begin{cases} i=1 & 0 \leq w \leq w_1 \\ i=2 & w_1 \leq w \leq w_2 \end{cases} \quad (2)$$

$$w_1 = \frac{1-b_2}{\alpha_1 - \alpha_2}, w_2 = \frac{b_2}{\alpha_2} \quad (3)$$

$$\beta_1 = \frac{f_t a_1 s}{E}, \beta_2 = \frac{f_t a_2 s}{E}$$

$$c = \frac{(1-b_2)(1-\beta_1)}{\beta_2 - \beta_1} \quad (4)$$

$$\mu = \frac{6M}{f_t b h^2}, \theta = \frac{hE\phi}{sf_t}, \alpha = \frac{d}{h} \quad (5)$$



$$0 < \theta \leq 1, \mu = \theta \quad (6)$$

$$1 < \theta \leq \theta_{I-II}$$

$$\alpha = 1 - \beta_1 - \sqrt{(1-\beta_1)\left(\frac{1}{\theta} - \beta_1\right)} \quad (7)$$

$$\mu = 4(1-3\alpha+3\alpha^2 - \frac{\alpha^3}{1-\beta_1})$$

$$\theta_{I-II} = 0.5(1-c + \sqrt{(1-c)^2 + \frac{c^2}{\beta_1 - 1}}) \quad (8)$$

$$\theta_{I-II} < \theta \leq \theta_{II-III} \quad (9)$$

6. $V1(Karihaloo) = 0.76 - 2.28.y_1 + 3.87.y_1^2 - 2.04.y_1^3 + 0.66/(1 - y_1)^2$ (41)

(1) .[5] Danhash et al

$y = \frac{a_0}{H}, y_1 = (a_0 + H)/(H + d_0)$ (22)

: 2-4 : -4
: -1-4

H a_0
0.5 0.1 a_0/H

13 :
Zhao [9]
Roesler [10]
16 Casuccio [12]
.Zhang [13]

(2)

Ft, E 10-80 [mm]

(1)

Concrete	Beam	Height H [mm]	Width B [mm]	Span S [mm]	Notch /height a0/H	Mass [kg]
C1 Danhash et al.	B1	150	80	600	0.33	16.5
	B2	150	70	600	0.37	15.7
	B3	150	68	600	0.37	14.5
	B4	150	80	600	0.30	15.3
	B5	100	80	600	0.53	15.3
	B6	150	80	600	0.53	15.9
Roseler	B1→3	150	80	600	0.33	17.3
	B4→6	250	80	1000	0.33	48
C40, C80 Zhang	All	100	100	400	0.1	9.2
Casuccio	all	105	75	400	0.5	7.5
Zhao SG3-6	all	300	120	1200	0.4	10
WG1 Zhao	B1	250	120	1000	0.40	73
	B2	300	120	1200	0.40	105
	B3	400	120	1600	0.40	188
LG1 Zhao	B1	400	240	1600	0.40	376
	B2	450	240	1800	0.40	476
	B3	500	240	2000	0.40	588
	B4	550	240	2200	0.40	711
SG1 Zhao	B1	300	120	1200	0.4	102
	B2	400	120	1600	0.4	182
Einsfeld	B1	76.2	38.1	400	0.50	7.4
	B2	152.4	38.1	400	0.50	7.5
	B3	304	38.1	400	0.50	7.6

(2)

Mix	Cement Kg/m ³	Gravel Kg/m ³	D _{max} [mm]	Sand Kg/m ³	Water Kg/m ³	W.R [Kg/m ³]	Fly ash	W/C	A.E %
C1Danhash et al.	400	1081	20	777	160	-	-	0.4	-
SG1 Zhao	196	1090	10	869	140	1.68	84	0.5	1.96
SG3 Zhao	240	1154	20	814	135	1.80	60	0.45	1.95
SG4 Zhao	309	1145	20	744	135	2.32	77	0.35	2.51
SG5 Zhao	420	1121	20	698	140	2.80	47	0.3	2.80
SG6 Zhao	168	1287	40	769	120	1.44	72	0.6	1.68
LG1 Zhao	159	1496	80	625	102	1.36	68	0.45	1.59
WG1 Zhao	159	1065	40	625	102	1.36	68	0.45	1.59
Roesler	290	1107	19	718	160	1.68W.R+ 1.65F ¹	88	0.55	0.24
Einsfield	420	992	9.5	860	149	11.5	S.P ² 11.5	0.31	-
Casuccio G18	263	1080	30	835	184	-	-	0.7	1.5
Casuccio G37	431	1060	30	765	149	4.2	-	0.35	2.5
Casuccio G48	452	960	30	875	155	4.8	-	0.34	3.5
Zhang C40	397	1065	10,16, 20,25	532	205	-	70	0.52	-
Zhang C80	450	1144	10,16, 20,25	572	150	-	SF ³ (50)	0.33	-

AE: Air entraining agent. 1: F is high range water reducer. 2: S.P is super plasticizer. 3: SF is silica fume.

.af -3 : -5

.a1,a2,b2 -4

.W_c -5 (W_c G_F)

. -6 -

.Kic -7

-8 .][9],[15

.CODc :

:

Kic

(3) .CODc

a1, a2, b2

:

-6 %1

tRienhard [11]

(S/D=4) :

MATLAB

-1

.G_F -2

CODc Kic

$$K(P, a) = \frac{3PS\sqrt{a}}{2D^2B} F_3 \left(\frac{a}{D} \right) \quad (23)$$

(4)

F3

.CODc Kic

$$F_3 \left(\frac{a}{D} \right) = \frac{1.99 - a/D (1 - \frac{a}{D}) [2.15 - 3.93 (\frac{a}{D}) + 2.7 (\frac{a}{D})^2]}{(1 + \frac{2a}{D}) (1 - \frac{a}{D})^{3/2}} \quad (24)$$

(3)

Mix	Beam	الخواص المادية			d_{max} [mm]	P.V	بارامترات التحليل العكسي			
		F'c [Mpa]	E [Gpa]	Ft [Mpa]			af [mm]	W= [mm]	Wc [mm]	G _F N/mm
SG1	B1	43.8	31.4	3.73	20	0.24	60.190	0.038	0.510	0.332
	B2				20	0.24	68.460	0.036	0.518	0.329
SG3	B1	50.9	35.7	3.45	20	0.24	79.610	0.059	0.330	0.269
SG4	B1	56.4	35.9	3.67	20	0.27	75.110	0.056	0.299	0.258
SG5	B1	50.2	41	3.42	20	0.30	76.460	0.048	0.250	0.237
SG6	B1	50.8	38.9	3.45	40	0.21	68.800	0.030	0.370	0.262
WG1	B1	40	33.6	3.51	40	0.18	56.100	0.033	0.277	0.287
	B2				40	0.18	71.040	0.053	0.482	0.315
	B3				40	0.18	61.740	0.029	0.546	0.331
LG1	B1	40	33	3.51	80	0.18	54.640	0.021	0.669	0.376
	B2				80	0.18	55.480	0.028	0.635	0.387
	B3				80	0.18	77.210	0.034	0.686	0.431
	B4				80	0.18	58.360	0.031	0.645	0.395
C1	B1	33.7	35	3.70	20	0.28	50.540	0.069	0.220	0.211
	B2				20	0.28	48.600	0.065	0.216	0.208
	B3				20	0.28	44.660	0.041	0.191	0.185
	B4				20	0.28	48.580	0.040	0.178	0.204
	B5				20	0.28	35.180	0.046	0.232	0.188
	B6				20	0.28	36.210	0.045	0.250	0.198
Roesler	B1	58.3	32	3.74	20	0.29	38.560	0.027	0.280	0.195
	B2				20	0.29	39.530	0.033	0.301	0.214
	B3				20	0.29	37.100	0.020	0.201	0.186
	B4				20	0.29	49.480	0.034	0.375	0.242
	B5				20	0.29	47.820	0.024	0.200	0.163
	B6				20	0.29	43.800	0.023	0.172	0.155
Casuccio	B1	18.1	27.1	3.40	30	0.27	18.670	0.014	0.194	0.106
	B2	37.5	33.1	4.10	30	0.29	19.560	0.015	0.175	0.132
	B3	48.4	39.9	5.30	30	0.30	15.370	0.010	0.171	0.136
Zhang C40	D10 B1	39.55	30	3.208	10	0.36	38.740	0.010	0.156	0.145
	D10 B2				10	0.36	31.020	0.015	0.106	0.135
	D16 B1	40.05	30	4.555	16	0.36	29.400	0.024	0.225	0.193
	D16 B2				16	0.36	20.570	0.014	0.227	0.177
	D20 B1	39.23	30	6.190	20	0.36	9.030	0.031	0.437	0.239
	D20 B2				20	0.36	16.060	0.019	0.374	0.219
	D25 B1	40.07	30	2.87	25	0.36	43.910	0.037	0.397	0.223
	D25 B2				25	0.36	46.870	0.055	0.433	0.217
Zhang C80	D10 B1	85.2	35	5.793	10	0.32	24.810	0.019	0.141	0.176
	D10 B2				10	0.32	29.160	0.026	0.168	0.199
	D16 B1	85.42	35	5.335	16	0.32	34.550	0.031	0.153	0.212
	D16 B2				16	0.32	29.180	0.025	0.155	0.172
	D20 B1	82.29	35	5.458	20	0.32	33.820	0.031	0.172	0.235
	D20 B2				20	0.32	33.820	0.031	0.172	0.235
	D25 B1	82.28	35	3.844	25	0.32	42.120	0.041	0.345	0.270
	D25 B2				25	0.32	41.660	0.042	0.293	0.232

concrete	CODc			Kic		(4)	
	F'c[Mpa]	Ft[Mpa]	d _{max} [mm]	Kic[Mpa.mm ^{0.5}]	CODc[mm]	P.V[m ³ /m ³]	
Zhao SG1	43.8	3.73	20	41.595	0.019	0.241	
Zhao SG3	50.9	3.45	20	43.841	0.025	0.240	
Zhao SG4	56.4	3.67	20	45.806	0.024	0.270	
Zhao SG5	50.2	3.42	20	43.481	0.021	0.297	
Zhao SG6	50.8	3.45	40	39.036	0.016	0.207	
Zhao WG1	40	3.51	40	31.970	0.018	0.184	
Zhao LG1	40	3.51	80	77.294	0.017	0.184	
Danhash et al. C1	33.7	3.7	20	21.067	0.019	0.277	
Roseler	58.3	3.74	20	26.944	0.014	0.193	
Cassucio1	18.1	3.4	30	10.395	0.006	0.106	
Cassucio2	39.55	4.1	30	12.912	0.007	0.132	
Cassucio3	48.4	5.3	30	14.996	0.005	0.136	
Zhang C4010	39.55	3.208	10	37.337	0.011	0.155	
Zhang C4016	40.05	4.555	16	41.670	0.010	0.185	
Zhang C4020	39.23	6.19	20	39.770	0.008	0.229	
Zhang C4025	40.07	2.87	25	48.479	0.021	0.270	
Zhang C8010	85.2	5.793	10	55.577	0.011	0.188	
Zhang C8016	82.42	5.335	16	57.772	0.013	0.192	
Zhang C8020	82.29	5.458	20	61.845	0.015	0.235	
Zhang C8025	82.28	3.844	25	52.170	0.016	0.251	

(5) : -7

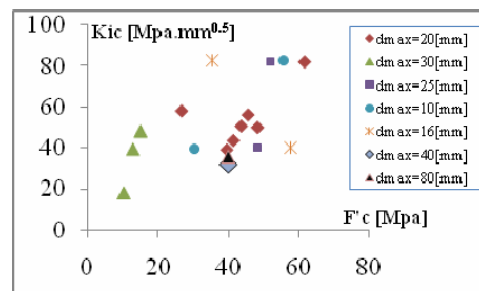
(CODc)

: F'c Kic

(5)

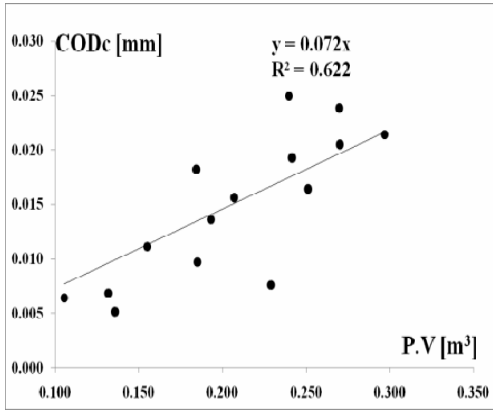
5 4 F'c Kic

(CODc)



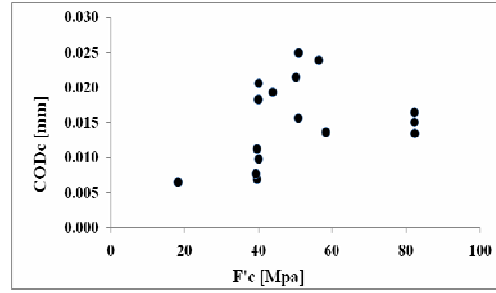
Kic (5)

F'c



(8)

P.V CODc



(6)

F'c CODc

(7)

.P.V Kic

: -8

-1

:

P.V

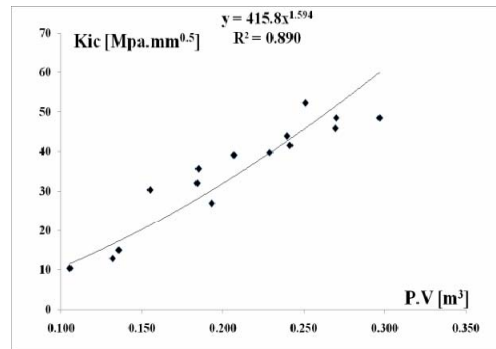
Kic

$$K_{ic} = 415.8 \cdot (P.V)^{-1.594} \quad (25)$$

-2

-3

F'c K_{ic}
(CODc) (Kic)



(7)

Kic P.V

(Kic)

(CODc)

(8)

(CODc)

.(P.V)

(P.V)

(CODc)

:

$$COD_c = 0.072 \cdot P.V \quad (5)$$

	h		A_{lig}
	k		a_0
	L		a
	M		B
	μ		b
	D		c
	m		d
	P.V		E
	S		F_t
	θ		F'_c
	θ_c		δ_0
	w		G_f
()	w_c	RILEM	$G_f(exp)$
	W_0		φ
	z		g
	a1, a2 ,b2		H_0

References *

- [1] Béton, C.E.-I.d., CEB-FIP Model Code 1990. Thomas Telford: London, UK. p. 437.
- [2] Casuccio, M., et al., 2008- Failure mechanism of recycled aggregate concrete. *Construction and Building Materials*, 2008. 22(7): p. 1500-1506.
- [3] Einsfeld, R.A. and M.S.L. Velasco, 2006- Fracture parameters for high-performance concrete. *Cement and Concrete Research*, 2006. 36(3): p. 576-583.
- [4] Elices, E. and Planas, J. 1996_ Fracture mechanics parameters of concrete: an overview, *Advanced Cement Based Materials*, 4, 116-127.
- [5] G.Danhash, M.Kousa, G.Wardeh. 2012_ "Investigating the size independent Fracture Energy of concrete using Inverse Analysis" *The journal of Damascus university*.
- [6] Hillerborg, A., M. Modéer, and P.-E. Petersson, 1976 -Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and Concrete Research*,. 6(6): p. 773-781.
- [7] Karihaloo, B.L. and P. Nallathambi, 1991- Notched beams test: model I fracture toughness, in *Fracture Mechanics Test Method for concrete*, S. Shah and A. Carpiteri, Editors., Chapman & Hall. p. chapter 1, pages 1-86.
- [8] Olesen, J.F. 2001_ Fictitious Crack Propagation in Fiber-Reinforced Concrete Beams. *Journal of Engineering Mechanics*. 127(3): p. 272-280.
- [9] Ostergaard, L., 2003- Early-Age Fracture Mechanics And Cracking of Concrete. Experiments and Modeling, in *Department of Civil Engineering*., Technical University of Denmark. p. 299.
- [10] RILEM 50-FMCDraft Recommendation, 1985- Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams. *Materials and Structures/Matériaux et Constructions*,. 18(4): p. 287-290.
- [11] Hans W.Reinhardt , Shilang Xu. 1999_ "Crack resistance based on the cohesive force in concrete". *Engineering Fracture Mechanics* 64 ,563-587.
- [12] Roesler, J., et al., 2007- Concrete fracture prediction using bilinear softening. *Cement and Concrete Composites*,. 29(4): p. 300-312.
- [13] Stang, H., 2000- Analysis of 3 points bending test , *Technical Report EU Contract - BRPR - CT98 - 813* , Report from Test and Design Methods for Steel Fiber Reinforced concrete.. p. 7 pages.
- [14] Ulfkjaer, J.P., S. Krenk, and R. Brincker, 1995- Analytical Model for Fictitious Crack Propagation in Concrete Beams. *Journal of Engineering Mechanics*,. 121(1): p. 7-15.
- [15] Wittmann, F., et al., 1988- Fracture energy and strain softening of concrete as determined by means of compact tension specimens, in *Materials and Structures*., Springer Netherlands. p. 21-32.
- [16] Zhao, Z., S.H. Kwon, and S.P. Shah, 2008- Effect of specimen size on fracture energy and softening curve of concrete: Part I. Experiments and fracture energy. *Cement and Concrete Research*, 38(8-9): p. 1049-1060.
- [17] Zhang, J., C. Leung, and S. Xu, 2010- Evaluation of fracture parameters of concrete from bending test using inverse analysis approach, in *Materials and Structures*. 2010, Springer Netherlands. p. 857-874.